AP Calc AB
Last test review (2)
Answers

① $\int x^2 e^{4x}\,dx$

sign	u & du	dv & v
+	x^2	e^{4x}
−	$2x$	$\frac{1}{4}e^{4x}$
+	2	$\frac{1}{16}e^{4x}$
−	0	$\frac{1}{64}e^{4x}$

$$\boxed{\frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + c}$$

② $$\int \frac{5x-1}{x^2-2x-15}\,dx$$
$(x-5)(x+3)$

$$\frac{A}{x-5} + \frac{B}{x+3} = \frac{5x-1}{(x-5)(x+5}$$

$$A(x+3) + B(x-5) = 5x-1$$

For $x = -3$:

$-8B = -16$

$B = 2$

For $x = 5$:

$8A = 24$

$A = 3$

$$\int\left(\frac{2}{x-5} + \frac{3}{x+3}\right)dx = \boxed{2\ln|x-5| + 3\ln|x+3| + c}$$

③ $$\int \frac{x^2+4x-18}{x^3-6x^2+9x}\,dx \qquad \frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}=\frac{x^2+4x-18}{x^3-6x^2+9x}$$

$x(x-3)(x-3)$

$$A(x-3)^2+Bx(x-3)+Cx = x^2+4x-18$$

For $x=3$:

$3C=3$

$C=1$

For $x=0$:

$9A=-18$

$A=-2$

For $x=1$, $A=-2$, $C=1$:

$4A-2B+C=-13$

$-8-2B+1=-13$

$B=3$

$$\int\left(\frac{-2}{x}+\frac{3}{x-3}+\frac{1}{(x-3)^2}\right)dx=\boxed{-2\ln|x|+3\ln|x-3|-\frac{1}{x-3}+c}$$

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④ $$\int x^3\ln(x)\,dx \qquad u=\ln(x) \quad dv=x^3dx$$

$$du=\frac{1}{x}dx \quad v=\frac{1}{4}x^4$$

$$\frac{1}{4}x^4\ln(x)-\int\frac{1}{4}x^4\cdot\frac{1}{x}\,dx = \frac{1}{4}x^4\ln(x)-\frac{1}{4}\int x^3\,dx$$

$$\frac{1}{4}x^4\ln(x)-\frac{1}{16}x^4+c$$

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⑤ $$\lim_{x\to 3}\frac{\sqrt{x+13}-4}{2x^2-18}=\frac{0}{0}$$

$$\lim_{x\to 3}\frac{\frac{1}{2}(x+13)^{-\frac{1}{2}}}{4x}=\frac{1}{8x\sqrt{x+13}}=\frac{1}{8(3)(4)}=\boxed{\frac{1}{96}}$$

⑥ $\int e^{2x} \sin(5x)\,dx$ $\qquad u = \sin(5x) \quad dv = e^{2x}\,dx$

$du = 5\cos(5x) \quad v = \frac{1}{2}e^{2x}$

$\frac{1}{2}e^{2x}\sin(5x) - \int \frac{5}{2}e^{2x}\cos(5x)\,dx$ $\qquad u = \cos(5x) \quad dv = \frac{5}{2}e^{2x}\,dx$

$du = -5\sin(5x) \quad v = \frac{5}{4}e^{2x}$

$$\frac{1}{2}e^{2x}\sin(5x) - \left[\frac{5}{4}e^{2x}\cos(5x) - \int -\frac{25}{4}e^{2x}\sin(x)\,dx\right]$$

$$\frac{1}{2}e^{2x}\sin(5x) - \frac{5}{4}e^{2x}\cos(5x) - \frac{25}{4}\int e^{2x}\sin(x)\,dx = \int e^{2x}\sin(5x)\,dx$$

$$+\frac{25}{4}\int e^{2x}\sin(x) \qquad + \frac{25}{4}\int e^{2x}\sin(5x)\,dx$$

$$\frac{1}{2}e^{2x}\sin(5x) - \frac{5}{4}e^{2x}\cos(5x) = \frac{29}{4}\int e^{2x}\sin(5x)\,dx$$

$$\int e^{2x}\sin(5x)\,dx = \boxed{\frac{4}{29}\left[\frac{1}{2}e^{2x}\sin(5x) - \frac{5}{4}e^{2x}\cos(5x)\right] + C}$$

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⑦ $\int \frac{5x^2 - 5x + 21}{x^3 + 7x}\,dx$ $\qquad \frac{5x^2-5x+21}{x(x^2+7)} = \frac{A}{x} + \frac{Bx + C}{x^2+7}$

$$A(x^2+7) + (Bx+C)x = 5x^2 - 5x + 21$$

$$Ax^2 + 7A + Bx^2 + Cx = 5x^2 - 5x + 21 \rightarrow (A+B)x^2 + Cx + 7A = 5x^2 - 5x + 21$$

$A+B = 5 \qquad C = -5 \qquad 7A = 21$

$3 + B = 5 \qquad\qquad A = 3$

$B = 2$

$$\int \left(\frac{3}{x} + \frac{2x}{x^2+7} - \frac{5}{x^2+7}\right)dx$$

$$\boxed{3\ln|x| + \ln(x^2+7) - \frac{5}{\sqrt{7}}\arctan\frac{x}{\sqrt{7}} + C}$$

⑧ $\int x^4 \cos(2x)\, dx$

sign	u & du	dv & V
+	x^4	$\cos 2x$
−	$4x^3$	$\frac{1}{2}\sin(2x)$
+	$12x^2$	$-\frac{1}{4}\cos(2x)$
−	$24x$	$-\frac{1}{8}\sin(2x)$
+	24	$\frac{1}{16}\cos(2x)$
−	0	$\frac{1}{32}\sin(2x)$

$$\boxed{\frac{1}{2}x^4\sin(2x) + x^3\cos(2x) - \frac{3}{2}\sin(2x) - \frac{3}{2}\cos(2x) + \frac{3}{4}\sin(2x) + c}$$

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⑨ $\int \frac{3x^3 - x^2 + 15x}{(x^2+3)^2}\, dx \longrightarrow \frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2} = \frac{3x^3 - x^2 + 15x}{(x^2+3)^2}$

$$(Ax+B)(x^2+3) + Cx + D = 3x^3 - x^2 + 15x$$

$$Ax^3 + Bx^2 + (3A+C)x + 3B + D = 3x^3 - x^2 + 15x$$

$A = 3 \qquad B = -1 \qquad 3A + C = 15 \qquad 3B + D = 0$

$9 + C = 15 \qquad -3 + D = 0$

$C = 6 \qquad D = 3$

$$\int \left(\frac{3x}{x^2+3} - \frac{1}{x^2+3} + \frac{6x}{(x^2+3)^2} + \frac{3}{(x^2+3)^2} \right) dx$$

$$\boxed{\frac{3}{2}\ln(x^2+3) - \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + \frac{3}{x^2+3} + \int \frac{3}{(x^2+3)^2}\, dx + c}$$

(10) $\int e^{3x}\cos(3x)\,dx$

$u = \cos(3x) \qquad dv = e^{3x}\,dx$

$du = -3\sin(3x)\,dx \quad v = \frac{1}{3}e^{3x}$

$\frac{1}{3}e^{3x}\cos(3x) - \int -e^{3x}\sin(3x)\,dx$

$u = \sin(3x) \quad dv = -e^{3x}\,dx$

$du = 3\cos(3x)\,dx \quad v = -\frac{1}{3}e^{3x}$

$\frac{1}{3}e^{3x}\cos(3x) - \left[-\frac{1}{3}e^{3x}\sin(3x) - \int -e^{3x}\cos(3x)\,dx\right]$

$\frac{1}{3}e^{3x}\cos(3x) + \frac{1}{3}e^{3x}\sin(3x) - \int e^{3x}\cos(3x)\,dx = \int e^{3x}\cos(3x)\,dx$

$+\int e^{3x}\cos(3x)\,dx \quad +\int e^{3x}\cos(3x)\,dx$

$\frac{1}{3}e^{3x}\cos(3x) + \frac{1}{3}e^{3x}\sin(3x) = 2\int e^{3x}\cos(3x)\,dx$

$$\boxed{\int e^{3x}\cos(3x)\,dx = \frac{1}{2}\left[\frac{1}{3}e^{3x}\cos(3x) + \frac{1}{3}e^{3x}\sin(3x)\right] + C}$$

⑪ $\lim_{x\to\infty} \frac{x^4}{e^x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{4x^3}{e^x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{12x^2}{e^x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{24x}{e^x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{24}{e^x} = \boxed{0}$

⑫ $\lim_{x\to 3^-} \frac{2x-6}{x^2-6x+9} = \frac{0}{0}$

$\lim_{x\to 3^-} \frac{2\cancel{(x-3)}}{(x-3)^{\cancel{2}}} = \frac{2}{x-3} = \frac{2}{0} = +\text{ or }-\ \infty$

$3^-: \ \frac{+}{-} = \boxed{-\infty}$